BC Calc Vector 3 Name: Key 2021 Block: Seat:

1. (NC) The position of a particle at any time $t \geq 0$ is given by $x(t) = t^2 - 2$, $y(t) = \frac{2}{3}t^3$.

   (a) Find the magnitude of the velocity vector at $t = 2$.

$$\frac{dx}{dt} = 2t \qquad \frac{dy}{dt} = 2t^2$$

$$\vec{v}(t) = \langle 2t, 2t^2 \rangle$$

$$\|\vec{v}(2)\| = \|\langle 4, 8 \rangle\| = \sqrt{4^2 + 8^2} = 4\sqrt{5}$$

   (b) Set up an integral expression to find the total distance traveled by the particle from $t = 0$ to $t = 4$.

$$\int_0^4 \sqrt{4t^2 + 4t^4}\, dt \approx 46.061$$

or

$$46.062$$

   (c) Find $\frac{dy}{dx}$ as a function of x.

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t^2}{2t} = t$$

   (d) At what time t is the particle on the y-axis? Find the acceleration vector at this time.

when vert. comp. is 0

$$\frac{2}{3}t^3 = 0$$

$$t = 0$$

$$\vec{v}(t) = \langle 2t, 2t^2 \rangle$$

$$\vec{v}(0) = \langle 0, 0 \rangle$$

$$\vec{a}(t) = \langle 2, 4t \rangle$$

$$\vec{a}(0) = \langle 2, 0 \rangle$$

2. (NC) An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time t with the velocity vector $v(t) = \langle (t+1)^{-1}, 2t \rangle$. At time $t = 1$, the object is at $(\ln 2, 4)$.

(a) Find the position vector.

$$\vec{s}(t) = \int \left\langle \frac{1}{t+1}, 2t \right\rangle dt$$

$$= \left\langle \ln|t+1| + C_1,\ t^2 + C_2 \right\rangle$$

Now $\ln(1+1) + C_1 = \ln 2$

so $C_1 = 0$

and $1 + C_2 = 4$ so $C_2 = 3$

$$\vec{s}(t) = \left\langle \ln|t+1|, t^2+3 \right\rangle$$

(b) Write an equation for the line tangent to the curve when $t = 1$.

$$\frac{dy}{dx} = \frac{2t}{\frac{1}{t+1}} = 2t(t+1)\Big|_{t=1} = 4$$

given $(\ln 2, 4)$ at $t=1$

$$y - 4 = 4(x - \ln 2)$$

or $y = 4x + 4 - \ln 16$

(c) Write ~~an equation for the line tangent to the curve~~ when $t = 1$.
What is the speed,

$$\sqrt{\left(\frac{1}{2}\right)^2 + 2^2} = \sqrt{\frac{17}{4}} = \frac{\sqrt{17}}{2}$$

How much has it travelled in the first second?

$$\int_0^1 \sqrt{\left(\frac{1}{t+1}\right)^2 + (2t)^2}\, dt$$

(d) At what time $t \geq 0$ does the line tangent to the particle at $(x(t), y(t))$ have a slope of 12?

$$2t(t+1) \geq 12$$

$$t^2 + t - 6 \geq 0$$

$$(t+3)(t-2) \geq 0$$

$$t = -3 \text{ or } t = 2$$

both work (if you remembered the ab. value!)

3. (Calc OK) A particle moving along a curve in the xy-plane has position $(x(t), y(t))$, with $x(t) = 2t + 3\sin t$ and $y(t) = t^2 + 2\cos t$, where $0 \le t \le 10$.
Find the velocity vector at the time when the particle's vertical position is $y = 7$.

$$\vec{v}(t) = \langle 2 + 3\cos t,\ 2t - 2\sin t \rangle$$

Find t when $y = 7$

$$t^2 + 2\cos t = 7$$

$$A = t = 2.9964952$$ (don't round yet! store it!)

$$\vec{v}(A) \approx \langle -0.968,\ 5.703 \rangle$$

or $\langle -0.968,\ 5.704 \rangle$

4. (Calc OK) A particle moving along a curve in the xy-plane has position $(x(t), y(t))$ at time t with $\frac{dx}{dt} = 1 + \sin(t^3)$. The derivative $\frac{dy}{dt}$ is not explicitly given. For any time t, $t \ge 0$, the line tangent to the curve at $(x(t), y(t))$ has a slope of $t + 3$. Find the acceleration vector of the object at time $t = 2$.

Since $\frac{dy}{dx} = \frac{dy/dt}{dx/dy}$, $\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dy} = (t+3)(1+\sin t^3)$

$$\frac{dx^2}{dt^2} = 3t^2 \cos t^3$$

$$\frac{dy^2}{dt^2} = (t+3)(3t^2\cos t^3) + (1 + \sin t^3)$$

$$\vec{a}(t) = \langle 3t^2 \cos t^3,\ (t+3)(3t^2\cos t^3) + (1+\sin t^3)$$

$$\vec{a}(2) \approx \langle -1.746,\ -6.740 \rangle$$ or
$\langle -1.746,\ -6.741 \rangle$

(Calc not really needed for most of this)

5. An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time t with $\frac{dx}{dy} = \cos(e^t)$ and $\frac{dy}{dt} = \sin(e^t)$ for $0 \le t \le 2$. At time $t = 1$, the object is at the point $(3, 2)$.

(a) Find the equation of the tangent line to the curve at the point where t = 1.

$$\left.\frac{dy}{dx}\right|_{t=1} = \frac{\sin e^1}{\cos e^1} = \tan e$$

$$y - 2 = \tan e\,(x-3)$$

or about

$$y = -0.4505(x-3) + 2$$

(b) Find the speed of the object at t = 1.

$$\left.\sqrt{(\cos e^t)^2 + (\sin e^t)^2}\,\right|_{t=1} = \sqrt{\cos^2 e + \sin^2 e} = 1$$

(c) Find the total distance traveled by the object over the time interval $0 \le t \le 2$.

$$\int_0^2 \sqrt{\cos^2 e^t + \sin e^t}\, dt = \int_0^2 dt$$

$$= t\Big|_0^2 = 2 - 0 = 2$$

(d) Find the position of the object at time $t = 2$.

$$x = 3 + \int_1^2 \cos e^t\, dt \approx 2.8957$$

$$y = 2 + \int_1^2 \sin e^t\, dt \approx 1.6759$$

$(2.895, 1.675)$ or $(2.896, 1.676)$

6. A particle moving along a curve in the xy-plane has position $(x(t), y(t))$ at time t with $\dfrac{dx}{dt} = \sin(t^3 - t)$ and $\dfrac{dy}{dt} = \cos(t^3 - t)$. At time $t = 3$, the particle is at the point $(1, 4)$.

(a) Find the acceleration vector for the particle at $t = 3$.

$$\vec{a}(t) = \langle (3t^2-1)\cos(t^3-t),\ -(3t^2-1)\sin(t^3-t) \rangle$$

$$\vec{a}(3) = \langle 26\cos 24,\ -26\sin 24 \rangle$$

about $\langle 11.028, 23.545 \rangle$ or $\langle 11.029, 23.545 \rangle$

(b) Find the equation of the tangent line to the curve at the point where $t = 3$.

$$\frac{dy}{dx} = \left.\frac{\cos(t^3-t)}{\sin(t^3-t)}\right|_{t=3} = \frac{\cos 26}{\sin 26} = \cot 26 \approx 0.84835$$

$$y = \cot 26\,(x-1)+4$$

(c) Find the magnitude of the velocity vector at $t = 3$.

$$\|\vec{v}(3)\| = \sqrt{\sin^2(26) + \cos^2(26)} = 1$$

(d) Find the position of the particle at time $t = 2$.

$$x = 1 + \int_3^2 \sin(t^3-t)\,dt \approx 0.932$$

$$y = 4 + \int_3^2 \cos(t^3-t)\,dt \approx 4.002$$

7. An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time t with $\frac{dy}{dt} = 2 + \sin(e^t)$. The derivative $\frac{dx}{dt}$ is not explicitly given. At $t = 3$, the object is at the point $(4, 5)$.

   (a) Find the y-coordinate of the position at time $t = 1$.

$$y = 5 + \int_3^1 2 + \sin(e^t)\, dt \approx 1.268 \text{ or } 1.269$$

$$\frac{dy}{dx} = -1.8 = \frac{-9}{5}$$

Typo ?

   (b) At time $t = 3$, the value of $\boxed{\frac{dy}{dt}}$ is -1.8. Find the value of $\frac{dx}{dt}$ when $t = 3$.

Since $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$, $\frac{dx}{dt} = \frac{dy}{dt} \cdot \frac{dx}{dy} = (2 + \sin(e^t))\left(\frac{-5}{9}\right)$

$$\left.\frac{dx}{dt}\right|_{t=3} = \frac{-10}{9} - \frac{5}{9}\sin(e^3) \approx -1.635 \text{ or } -1.636$$

   (c) Find the speed of the object at time $t = 3$.

$$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \approx 3.368$$